Default Option and the Cross-Section of Stock Returns

Assaf Eisdorfer **Amit Goyal** **Alexei Zhdanov***

December 2014

Abstract

We treat common stocks as options and construct a valuation model that takes into account the value of the option to default (or abandon the firm). The long-short strategy that buys stocks that are classified as undervalued by our model and shorts overvalued stocks generates an annual 4-factor alpha of about 11% for U.S. stocks. The model's performance is higher for stocks with high value of default option, such as distressed or highly volatile stocks. We also argue that investors' inability to value the default option properly creates valuation uncertainty and favorable conditions for return anomalies. We find that distress and idiosyncratic volatility anomalies are concentrated among most misvalued stocks (as classified by our model). The results are robust to various subsamples and return horizons.

*Assaf Eisdorfer is from University of Connecticut, email: Assaf.Eisdorfer@business.uconn.edu, Tel: 860-486-4485; Amit Goyal is from Swisss Finance Institute at the University of Lausanne, email: Amit.Goyal@unil.ch, Tel: +41-21-692-3676; and Alexei Zhdanov is from PanAgora Asset Management, email: azhdanov@panagora.com, Tel: 617-439-6383. Amit Goyal would like to thank Rajna Gibson for her support through her NCCR-FINRSK project. We thank Yakov Amihud, Malcolm Baker, John Campbell, Clifton Green, Amiyatosh Purnanandam (our AFA discussant), Norman Schürhoff, and seminar participants at Bar-Ilan University, Emory University, HKUST, NTU, NUS, University of Connecticut, Interdisciplinary Center Herzliya, University of Pireaus, and University of Warwick for helpful comments. All errors are our own.

1 Introduction

It has long been recognized in the finance literature that equity of a firm with debt in its capital structure is analogous to a call option written on the assets of the firm. The title of the seminal paper by Black and Scholes (1973) reflects the applicability of their model to the valuation of corporate debt and equity. Today, nearly every corporate finance textbook (see, for example, Brealey, Myers, and Allen (2011)) discusses the option-based approach to value equity and debt. An interesting question is whether analysts and investors incorporate this option-based approach in their equity valuation.

A key characteristic of corporate equity is the default option. Standard stock valuation techniques, such as multiples-valuation or discounted cash flow, ignore the option to default. Using these techniques, therefore, can lead to misvaluation, especially among stock with relatively high value of default option.[1] We build a structural model that takes into account the value of the option to default (or abandon the firm) and examine whether this option-based valuation model can predict future stock returns.

Option pricing models have been employed in the literature to gauge the probability of default and to value corporate bonds given the value of equity (see for example, Merton (1974), Geske (1978), and Delianedis and Geske (2003), among others). We, instead, use an option pricing model to value the equity itself. Our model uses standard features of structural models – stochastic cash flows, fixed costs, and debt. We also allow for endogenous default, different tranches of debt with different maturities, and additional costs of financial distress. While our model does not incorporate many aspects that have received considerable attention in corporate finance like investments or managerial entrenchment (see Ozdagli (2010) for a more carefully calibrated model of default), it is specifically tailored to value the default option. To the best of our knowledge, our paper is the first to employ a structural option pricing model of endogenous default on a large cross-section of stocks to value corporate equity.

We start our analysis by sorting all stocks every month into ten equal-sized portfolios according to the ratio of the model value to market value of equity; that is, ratios higher (lower) than one indicate under- (over-) valuation according to our model. We find that most misvalued stocks, either over- or undervalued, are smaller, more volatile, and less liquid, have fewer analyst coverage with higher analysts' forecast dispersion, and have lower institutional ownership; indicating that these stocks are the most difficult to value. Excess returns on

[1] It may seem that ignoring the optionality will always lead to undervaluation of equity. We do not make the strong claim that investors are unaware of the possibility of default by equity. Our conjecture is only that the investors do not value the resulting option properly. In other words, standard valuation techniques, by employing more crude proxies of this optionality, lead to misvaluation (under or over) of equity.

these sub-categories of stocks show patterns consistent with our valuation model. Monthly excess return for overvalued decile of stocks is 0.51% (4-factor alpha of −0.24%) while that for undervalued decile stocks is 1.13% (4-factor alpha of 0.65%). The long-short strategy that buys stocks that are classified as undervalued by our model and shorts overvalued stocks, thus, generates an annualized 4-factor alpha of about 11%. These differences are economically large and statistically significant. These results are also robust to various sub-samples and return horizons, and are confirmed using Fama and MacBeth (1973) regressions.

To explore the role of the default option more directly, we investigate how the returns generated by the model vary across stocks with characteristics related to default option. The first characteristic is the extent of financial distress. Distressed stocks have high levels of debt and a substantial probability of default that makes the analogy between equity value and a call option particularly relevant. We measure financial distress using the model proposed by Campbell, Hilscher, and Szilagyi (2008, henceforth CHS). The second characteristic is market capitalization; empirical evidence and conventional wisdom suggest that small companies are more likely to default. The third characteristic is financial leverage as there is a correlation between higher probability of default and leverage. The fourth and the last characteristic is stock return volatility. Option pricing theory shows that the value of option increases with the volatility of the underlying asset.

We sort all stocks by each characteristic into five equal-sized quintile, and then double-sort all stocks within each quintile into five equal-sized quintiles according to the model/market value ratio. Calculating the fraction of the default option value in the total model-equity value shows a clear relation between the importance of the default option and each of the characteristics. Most notably, for the top CHS quintile (most distressed stocks), the option to default accounts on average for 42.5% of equity value, compared to only 21.3% for the least distressed stock quintile. The difference in default option fraction between the two extreme size quintiles is 10%, between the two extreme leverage quintiles is 14%, and between the two extreme volatility quintiles is also 14%. These relations justify the choice of these characteristics, and also support the reliability of our model.

The returns generated by the model exhibit a clear pattern across the distress-based portfolios. Within the top distress quintile, undervalued stocks earn monthly 4-factor alpha higher by 1.32% than those by overvalued stocks. The equivalent difference within the bottom distress quintile is only 0.23% a month. The model's returns are also much higher among highly volatile stocks; 4-factor alpha of under/over value long-short strategy is 1.25% for the top volatility quintile, while is reduced to 0.50% among the least volatile stocks. The effect of firm size however is much weaker and not always monotonic. The model's 4-factor alpha is 0.66% for small firms and 0.48% for large firms. This is consistent with the fairly low difference between default option fractions of small and large stocks. Finally, portfolio returns for leverage quintiles are not consistent with our expectations as the difference in alpha for under- and over-valued stocks is slightly higher for low leverage stocks than it is for high leverage stocks.

The positive effect of firm characteristics, especially distress and volatility, on the model's performance strongly suggests that the option to default is a primary driver in the predictive ability of the model over future stock returns. To verify this finding we conduct the following test. We derive our model's equity values while shutting down the option to default, and we use these values to re-sort and calculate the returns within each characteristic-quintile. The model's performance is substantially weaker without the default option. The model's 4-factor alpha is reduced from 1.32% to 0.57% for most distressed stocks, and from 1.25% to 0.43% for most volatile stocks. These reductions provide further indication to the importance of default option in our model's estimates, and more generally, are consistent with the conjecture that option to default is hard to estimate, leading thus to stock mispricing.

The apparent difficulty to incorporate appropriately the default option in stock valuation creates uncertainty for the pricing of individual stocks, and as a result can generate favorable conditions for various return anomalies to persist.[2] While we are silent as to the exact mechanism(s) that drive asset pricing anomalies, we argue that anomalies are more likely to exist amongst stocks whose true values are unknown to the majority of investors. Deviations from true values are more likely to stay unnoticed and not necessarily arbitraged away. Our next hypothesis is that certain return anomalies associated with default option are concentrated in stocks that are the most misvalued. Our valuation model allows us to separate fairly valued versus misvalued stocks. We expect, therefore, that these anomalies are stronger in stocks that are the most misvalued according to our model.

To verify our conjecture, we sort all stocks into terciles based on whether they are fairly valued or misvalued (where the relative valuation is determined by the comparison of market value and model value). We then look at the strength of anomalies associated with default option (distress, leverage, and idiosyncratic volatility) in these terciles. The results are generally consistent with our conjecture. The 4-factor alpha to the low-high distress portfolio are

[2] Anecdotal evidence suggests that even top equity analysts do not recognize the default-like features of equity. We studied analyst reports on Ford Motor around late 2008 to early 2009. Ford was in deep financial distress at that time and the option to default was in-the-money. Yet there is no evidence that analysts from top investment banks incorporated that option value in their analysis. For example, Société Générale based its price estimate on the long-term enterprise-value-to-sales ratio, while Deutsche Bank and JP Morgan used enterprise value over EBITDA ratio. In addition, Deutsche bank used a discount rate of 20%, and Credit Suisse used a DCF model with a "big increase" in the discount rate. These different approaches result in very different values. For example, the JP Morgan target price for Ford in late October is \$2.43 per share, while the Credit Suisse target price is \$1.00 per share.

0.82% (0.56%) in misvalued (fairly valued) stocks; the excess returns to the low-high idiosyncratic volatility portfolio are 1.08% (0.34%) in misvalued (fairly valued) stocks. However, value gap is not a factor at all in explaining the leverage anomaly. These return patterns are confirmed using Fama-MacBeth regressions, and moreover, are significantly weaker when eliminating the option to default in model valuation. We conclude that the inability of investors to value default option correctly contributes significantly to the prevalence of these anomalies.

An interesting question raised by our study is why do investors not use option valuation models to value stocks. One potential explanation is the complexity involved in implementing such models. We hypothesize that many investors, especially retail investors, do not possess the necessary skills to implement such a model. This conjecture is consistent with the evidence presented by Poteshman and Serbin (2003) who document that investors often exercise call options in a clearly irrational manner, suggesting that it is hard for certain types of investors to understand and value options correctly (Poteshman and Serbin find that this is particularly true for retail investors; traders at large investment houses do not exhibit irrational behavior.) Furthermore, Benartzi and Thaler (2001) show that when faced with a portfolio optimization problem, many investors follow naïve and clearly suboptimal strategies again suggesting that investors fail to fully understand more sophisticated models. Hirshleifer and Teoh (2003) argue that limited attention and processing power may lead investors to ignore or underweight information that is important for an option-based model to produce unbiased valuations (for example, one of key inputs in our model is volatility of sales).

We would like to emphasize that our valuation model does a good job of relative valuation by separating under- and overvalued stocks, but does not necessarily capture the fundamental value of stocks. For example, amongst the stocks classified as fairly valued by our model, distressed stocks earn much lower returns. This implies that our valuation model still misses common factors that could explain the overall low/negative returns of distressed stocks. Explaining this low-return puzzle is outside the scope of our paper. Instead, we focus on why the return anomalies are the strongest among misvalued stocks.

We reiterate that the power of our model is in the valuation of the option to default and/or shut down the firm. For stocks far from the default boundary (e.g., stocks with high cash flows, low volatility, and low leverage ratios), normal valuation techniques are still adequate and not much may be gained by using our model for such stocks. This is confirmed by our empirical results. The performance of the long-short strategy based on our valuation model deteriorates when applied to firms with low value of default option. Finally, while our model can also be used to price corporate debt, the objective of our study is only to use it to value equity, and study the impact of default option valuation on stock returns; debt valuation is outside of the scope of our paper.

2 Valuation model

A key characteristic of corporate equity is the default option. One source of difficulty in valuing equity, therefore, may come from the necessity of using an appropriate model to account for the value of the option to default. Any valuation model that fails to properly value this option is going to produce values that are further away from fundamental value than a model that accounts for this option.

Option pricing structural models have been employed in the literature to gauge the probability of default and to value corporate bonds given the value of equity. Our objective is to deploy an option pricing model to perform valuation of equity. As we explain in detail later in this section, our option-pricing based approach scores over the traditional approach on two fronts. First, we are better able to estimate future cash flows by explicitly accounting for the exercise of the default option by the equityholders. Thus, our model accounts for the truncation in cash flows—at very low states of demand when cash flows are sufficiently negative it is optimal to exercise the default option rather than to continue to operate the firm. This optionality is missed by commonly employed valuation methods. Second, the estimation of time variation in discount rates is typically a difficult task; it becomes even more difficult for firms with high default risk where any small change in firm value can significantly change the risk of equity. The option-pricing approach bypasses this problem by conducting the valuation under a risk-neutral measure.[3]

Of course, the central insight that the equity of a firm with debt in its capital structure is analogous to a call option written on the assets of the firm dates back to the seminal paper by Black and Scholes (1973). While nearly every corporate finance textbook discusses the option-based approach to value equity and debt, academic research on using these models to perform equity valuation is sparse. Most of the studies perform valuation of some specific types of companies, such as internet or oil companies, in a real options framework (see Moon and Schwartz (2000) for a an example).[4] By contrast, we implement our model on the entire cross-section of stocks.

[3] A necessary assumption for this approach to work is that there exists a tradeable asset in the economy whose price is perfectly correlated with the stochastic process that drives the dynamics of the cash flows.

[4] One notable exception is Hwang and Sohn (2010). They test predictability of returns using valuations derived from Black and Scholes model on a large cross-section of companies. However, the abandonment option is not explicitly modeled in their approach.

2.1 Model

We assume that the cash flows of a firm are driven by a variable x that reflects stochastic demand for the firm's products. The firm incurs fixed costs and uses debt and, therefore, has contractual obligations to make coupon and principal payments to its debtholders. We also assume that a firm with negative free cash flow incurs an additional proportional expense η. This extra cost reflects expenses that a financially distressed firm has to incur in order to maintain healthy relationship with suppliers, retain its customer base, deal with intensified agency costs like the under-investment problem, or the additional costs of raising new funds to cover for the short fall in cash flows. The free cash flow to equityholders of firm i is then given by:

$$CF_{it} = (x_{it} - I_{it} - F_i)(1 - \tau) + \tau Dep_{it} - Capex_{it} + \eta \mathbf{1}_{(x_{it} - I_{it} - F_i)(1-\tau) + \tau Dep_{it} < 0)} - D_{it}, \quad (1)$$

where x_{it} is the state variable of firm i at time t, I_{it} is the total interest payments to debtholders due at time t, D_{it} is the principal repayment due at time t, F_i is the total fixed cost that the company incurs per unit of time (i.e one year), τ is the tax rate, τDep_{it} is the tax shield due to depreciation expense, $Capex_{it}$ is capital expenditures, and $\mathbf{1}_{(\cdot)}$ is an indicator variable. We further assume that x_{it} follows a geometric Brownian motion under the physical measure with a drift parameter $\mu_{i,P}$ and volatility σ:

$$\frac{dx_{it}}{x_{it}} = \mu_{i,P} dt + \sigma_i dW_t. \quad (2)$$

Default is endogenous in our model similar to the majority of structural models (see, for example, Leland (1994)). The equity holders are endowed with an option to default which they exercise optimally; they default if continuing to operate the firm results in a negative value. In our model default occurs when cash flow to equity holders is sufficiently negative.[5] Note that the presence of the fixed cost component, F_i, means that the equityholders may decide to shut down operations and abandon the firm if the cash flow turns sufficiently negative even when the firm is debt-free. The option to exit is valuable even for an all-equity firm as long as F_i is positive.

Stockholders maximize the value of equity (we abstract from any potential conflicts of interest between managers and stockholders). The value of equity, V_0, given the initial state

[5] Note that we implicitly assume that the equity holders of a firm with negative free cash flow may continue to inject cash (issue new equity) into the firm (unless they decide to default), but it is costly do so and this cost is reflected in the parameter η. This assumption is common in structural credit risk and capital structure models. Setting η equal to infinity would result in immediate default as soon as the cash flow to equity holders turns negative.

variable x_0 is equal to the expected present value of future cash flows under the risk-neutral measure discounted by the risk-free rate r:

$$E_{i0}(x_0) = \sup_{T_{x_d(t)}} \mathbf{E}^Q_{x_0} \int_0^{T_{x_d(t)}} e^{-rt} CF_{it} dt, \tag{3}$$

where $x_d(t)$ is the optimal default boundary and $T_{x_d(t)}$ is a first-passage time of the process x to the boundary $x_d(t)$.[6] The default boundary is a function of time because debt has final maturity and coupon and principal payments are allowed to vary over time. The equity value can be decomposed into the value that would accrue to equityholders should they be forced to operate the firm forever (the discounted cash flow component) and the value of the default (abandonment) option:

$$\text{Default option} = \sup_{T_{x_d(t)}} \mathbf{E}^Q_{x_0} \int_0^{T_{x_d(t)}} e^{-rt} CF_{it} dt - \mathbf{E}^Q_{x_0} \int_0^{\infty} e^{-rt} CF_{it} dt \ \geq 0. \tag{4}$$

Equation (4) shows two fundamental differences between our valuation approach and traditional valuation methods. First, we discount cash flows to equityholders only up until the stopping time $T_{x_d(t)}$. This stopping time is determined as the outcome of the optimization problem of the equityholders and results from the optimal exercise of the option to default. By contrast, the usual valuation methods implicitly assume an infinite discounting horizon and ignore that option (value only the first term on the right-hand side of first line of equation (4)). Second, we use the risk-free rate and discount payouts to shareholders under the risk-neutral measure, while the standard valuation methods perform discounting under the physical measure. This also distorts valuations because risk and the appropriate discount rate under the physical measure varies significantly as the firm moves in and out of financial distress. In other words, as is well-known, one cannot price an option by expectation under the physical measure.

[6]We assume that the absolute priority rule (APR) is enforced and the shareholders receive zero payoff upon default. Deviations from the absolute priority rule and non-zero value of equity in default would induce higher default option values and higher probability of default. See Garlappi and Yan (2011) for equity valuation when APR is violated. In their model, APR violations make distress stocks less risky as the firm approaches the default boundary; given the relatively safe payoff in default distress stocks betas go down. However, in our sample distress stocks appear riskier than the rest of the universe both in terms of their betas and volatility of returns.

2.2 Implementation

We use both annual and quarterly COMPUSTAT data items as inputs to the model. Structural models typically use either earnings or the unlevered firm value as a state variable that drives valuation.[7] While both cash flows and earnings seem reasonable candidates, they pose implementation issues. Earnings is an accounting variable which may not be directly related to valuation. Cash flow data, on the other hand, is often missing from quarterly COMPUSTAT data making it difficult to compute cash flow volatility. Furthermore, cash flows are subject to one-time items such as lump-sum investments, and therefore the current value of cash flows may not necessarily be representative of its evolution in the future. To smooth out these potential short-term variations in cash flows, we use gross margin (defined as sales less costs of goods sold) as a proxy for the state variable x. Hence, our state variable for firm i at time t is defined as:

$$x_{it} = Sales_{it} - COGS_{it}, \tag{5}$$

where $Sales_{it}$ is the annual sales and $COGS_{it}$ is the cost of goods sold. There is a lot of short-term variation in capital expenditures and depreciation. In order to reduce this noise, we compute industry averages for different distress categories of stocks. We use 2-digit SIC codes for industry average and distress quintiles based on CHS (2008) distress measure (Details on these calculations are given in Appendix A). Thus, we compute the average $Capex/Sales$ ratio for the 2-digit SIC industry/distress quintile over the last three years, $\overline{CSR}_{t-3,t}$, and use this ratio and current sales of firm i to proxy for firm i's capital expenditures:

$$Capex_{it} = Sales_{it} \times \overline{CSR}_{t-3,t}. \tag{6}$$

We model depreciation in a similar way:

$$Dep_{it} = Sales_{it} \times \overline{DSR}_{t-3,t}, \tag{7}$$

where $\overline{DSR}_{t-3,t}$ is the average depreciation to sales ratio for the 2-digit SIC industry/distress quintile over the last three years. We use selling, general, and administrative expenses (COMPUSTAT item $XSGA$) as a proxy for the fixed costs, F_i.

We assume that firms issue two types of debt: short-term and long-term debt, but the model can incorporate any arbitrary maturity structure of debt. We use COMPUSTAT annual items DLT (long-term debt) and $DLCC$ (debt in current liabilities) as proxies for

[7] See Goldstein, Ju, and Leland (2001) for a criticism of the use of unlevered firm value.

company's long- and short-term debt. We further assume that the short-term debt matures in one year, while the long-term debt matures in five years. Since the coupon rate of debt presumably depends on a company's default likelihood, we model the coupon rate on the long-term debt as the sum of the risk-free rate and the actual yield on debt with a corresponding credit rating. We use the sum of the average between the T-bill rate and the 10-year T-note rate as the risk-free rate. For credit spread rating, we first divide the firms into quintiles based on CHS (2008) distress measure. We then use yields on AAA-rated, BBB-rated, and BBB-rated bonds+2% for distress quintiles 1-2, 3-4, and 5, respectively. We further assume that in year five, after the long term debt is paid off (if the firm has not defaulted before), the firm refinances its debt to match the industry average leverage ratio. Details on the refinancing procedure are provided in Appendix B.

To model the growth rate of x_{it} under the physical measure we use the standard approach discussed in many corporate finance textbooks (see, for example, Brealey, Myers, and Allen (2011)). We first posit that capital expenditures generate growth. Thus, $Capex_{it}$ at time t results in an increase in after-tax cash flows by $Capex_{it}R_A$ at $t+1$ where R_A is the after-tax return on assets. Thus:

$$\begin{aligned}&(1-\tau)(Sales_{it+1} - COGS_{it+1}) + \tau Dep_{it+1} = \\ &\quad (1-\tau)(Sales_{it} - COGS_{it}) + \tau Dep_{it} + Capex_{it} \times R_A \, .\end{aligned} \tag{8}$$

Assuming that both Capex and depreciation are proportional to sales and using equations (6) and (7), we get:

$$\mu_{i,P} = \frac{\overline{CSR}_{t-3,t} R_A}{(1-\tau)GM_{it} + \tau \overline{DSR}_{t-3,t}}, \tag{9}$$

where GM_{it} is the gross margin ratio:

$$GM_{it} = \frac{Sales_{it} - COGS_{it}}{Sales_{it}}.$$

We assume that the gross margin ratio as well as the capex to sales ratio and depreciation to sales ratios for firm i remain constant in the future. The drift of x_{it} under the physical measure is then given by $\mu_{i,P} = R_A - DY_{i,P}$, while the growth rate under the risk-neutral measure is given by $\mu_{i,Q} = r - DY_{i,Q}$, where DY is the dividend yield and r is the risk-free rate. Since the dividend yield is the same under both measures, $DY_{i,P} = DY_{i,Q}$, it follows that:

$$\mu_{i,Q} = r - R_A + \mu_{i,P}. \tag{10}$$

To measure the return on assets R_A, we calculate the cost of equity using CAPM. We estimate firms' betas over the past three-year period and then average across all firms in the same 2-digit SIC industry that also fall in the same distress quintile based on the CHS (2008) measure of financial distress.[8] We model cost of debt using our assumptions for coupon rates as described above by indexing it to current yields on bonds with various credit ratings. The return on assets, R_A is then equal to the weighted average of the cost of equity capital and the cost of debt.

We proxy σ by the annualized quarterly volatility of sales over the last eight quarters. If quarterly sales are not available in COMPUSTAT, we use the average quarterly volatility of sales of the firms in the same 3-digit SIC industry over the last eight-quarter period. We use volatility of sales as opposed to volatility of x_{it} in equation (2) because we believe it better reflects the volatility of the underlying demand-driven stochastic process, which drives valuation in structural models like ours. Using volatility of x_{it} instead would capture some short-term variations in the costs of goods sold which are not related to the underlying economic uncertainty and therefore should not affect the value of the option to default.[9] We use 35% for the corporate tax rate, τ, while we set the distress costs, η, to 15%.[10] The inputs to the model are summarized in Table 1.

A potential minor problem with our approach is that some companies have negative current values of x_{it}. A vast majority of such companies are financially distressed. On average 5% of the companies have negative gross margin. However, this percentage increases to 16% amongst the top quintile of most distressed companies. Since we cannot assume geometric growth for such companies, we assume, instead, that x_{it} follows an arithmetic Brownian motion until the moment when it reaches the value equal to its annualized standard deviation (of course, before the company defaults), at which point we assume that x_{it} begins to grow geometrically. We obtain qualitatively similar results by ignoring these observations. Finally, we employ a standard binomial numerical algorithm to determine both the optimal default boundary and the value of equity in equation (3). Further numerical details on the implementation of our procedure are provided in Appendix B.

[8] We differentiate between firms in different distress categories because expected returns to claimholders vary depending on the degree of distress. We thank the referee for suggesting this approach.

[9] We get similar results using volatility of $Sales_{it} - COGS_{it}$.

[10] Weiss (1990) estimates the direct costs of financial distress to be of the order of 3% of firm value, Andrade and Kaplan (1998) provide estimates between 10% and 23%, while Elkamhi, Ericsson, and Parsons (2011) use 16.5% in their analysis. Our results are insensitive to these variations in η.

3 Model performance

We perform our valuation on the entire universe of stocks. Then, each month we sort all stocks into deciles according to the ratio of the equity value implied by our valuation model to the actual equity value. Decile one contains the most overvalued stocks while decile ten consists of the most undervalued stocks. The portfolios are value-weighted and held for one subsequent month.[11] This valuation sort is similar in spirit to scaling the market price in order to predict returns (Lewellen (2004)). While the most usual scaling variable is the book value, some studies use model implied valuation as a scaling variable. For example, Lee, Myers, and Swaminathan (1999) use the ratio of residual income value to market value to predict future returns. Our approach, while using a different valuation method, is similar in its use of the sorting variable.

We report the characteristics of these portfolios in Table 2. In addition to size, market-to-book, market beta (calculated using last three years of data), past six-month return, and standard deviation of daily stock returns, we also report the percentage of firms reporting negative earnings, number of analysts, the standard deviation of their forecasts, equity issuance, institutional ownership, and two proxies for liquidity, namely share turnover and Amihud's (2002) illiquidity measure. Accounting and stock return data are from CRSP and COMPUSTAT and all analyst data are from IBES. For each characteristic, we first calculate the cross-sectional mean and median of each portfolio. The table then reports the time-series averages of these means and medians. We exclude observations in the top and the bottom percentiles in calculating the means and medians. We include all common stocks, although our results are robust to the exclusion of financial stocks. The sample period for our study is 1983 to 2012 as the coverage of quarterly COMPUSTAT data is sparse before this date.

In unreported results, we find that median model value is 7.6% higher than the market value. This suggests that on average stocks are undervalued (consistent with investors ignoring or placing less emphasis on the default option). As expected, the median spread increases at times when credit spreads are high (10.4% vs. 5.4%) and also in recessions vs. expansions (11.6% vs. 7.2%), i.e. when default options are more valuable. The spread is also higher for stocks with low institutional ownership. It is indeed likely that more sophisticated institutional investors are in a better position to value default option and are more likely to buy undervalued stocks, for which the model-to-market ratio is relatively high.

Table 2 shows that the most misvalued stocks (over- or under-valued) in the extreme deciles are smaller, more volatile, less liquid (especially undervalued stocks), have fewer analyst coverage with higher analysts' forecast dispersion, and have lower institutional own-

[11] In unreported results, we find even stronger results using equally-weighted portfolios.

ership than more fairly valued stocks. While these observations are not especially surprising as these are presumably the characteristics of stocks that are the most difficult to value, these results do provide a first indication that our model successfully detects stocks whose market values move further away from fundamental values. The results further show that most overvalued stocks (decile one) have, unsurprisingly, higher market-to-book ratios than most undervalued stocks (decile ten), which also explain their higher market beta. Decile one stocks also show higher past returns and issue more equity than decile ten stocks. These equity issuance patterns are consistent with our valuation model under the additional assumption that managers of these firms understand true valuations and time the market in issuing equity.

We have conjectured that the stocks in the extreme deciles are the most misvalued by the market, apparently due to the market's inability to value the default option correctly. We check whether the stocks in these deciles do, in fact default more often than more fairly valued stocks. We calculate the fraction of stocks that default based on CRSP's delisting codes associated with poor performances, such as bankruptcy, liquidation, dropping due to bad performances, etc. In unreported results, we find that the average default rate of stocks in deciles one and ten is 6.5%, 10.9%, and 14.5% in one-, two-, and three-years after portfolio formation, respectively (cf. default rate of stocks in decile R5 is 1.1%, 2.2%, and 3.4%). These statistics provide further indication that the misvaluation picked up by our model is related to default option.

3.1 Portfolio returns of stocks sorted on model valuation

We proceed to check the efficacy of our valuation model by calculating returns of the ten portfolios. Table 3 reports the value-weighted monthly returns on each portfolio as well as the returns to the hedge portfolio that is long the most undervalued firm portfolio (decile ten) and short the most overvalued firm portfolio (decile one). In addition to reporting the average return in excess of the risk-free rate, we also report the alphas from one-, three-, and four-factor models. The one-factor model is the CAPM model. We use Fama and French (1993) factors in the three-factor model. These factors are augmented with a momentum factor in the four-factor model. All factor returns are downloaded from Ken French's website. All returns and alphas are in percent per month and numbers in parentheses denote the corresponding t-statistics.

Table 3 shows that returns and factor-model alphas are generally monotonically increasing when one moves from decile one to decile ten. The hedge portfolio has excess returns of 0.63% per month (t-statistic=2.05). Factor model alphas display patterns consistent with excess returns and characteristics of stocks shown previously in Table 2. For example, since decile ten stocks are, on average, smaller and have lower market-to-book ratios than decile one stocks, the 10−1 portfolio has lower 3-factor alpha at 0.47% than CAPM alpha at 0.79%. At the same time, since past returns for decile ten stocks are lower than those for decile one stocks, the 4-factor alpha of the long-short portfolio is higher at 0.88% (t-statistic=3.54). Regardless of the risk correction, the alphas of 10−1 portfolio are economically large and statistically significant. The significance of the three- and four-factor model alphas also emphasizes that our valuation model is more than just a sort on traditional value measures such as market-to-book.[12]

Since our holding period is only one month, we check the robustness of these results to the inclusion of a short-term reversal factor. Alpha from this alternate five-factor model is similar to that from a four-factor model; the five-factor alpha of the 10−1 portfolio is 0.82% (t-statistic=3.37). We also calculate a five-factor model with an additional liquidity factor of Pástor and Stambaugh (2003). The alpha from this model is even higher than the four-factor alpha at 0.94% (t-statistic=3.50). We also check whether these returns can be explained by a volatility factor. We find that the loading of the 10−1 portfolio return on changes in VIX (proxy for volatility factor) is small and statistically insignificant.

We further examine the robustness of the results to different subsamples and return horizons in Table 4. To reduce the clutter in the table, we report only the 4-factor alphas for each portfolio. To facilitate comparison with the main results, we also report the full-sample results in the first row of the table. We consider three different kinds of subsamples. The first simply tabulates results for the months of January versus the rest of the months. The second considers different states of the economy. We use NBER recession dummy as an indicator of the health of the economy for this exercise. Third, we consider calendar patterns in our results by separately tabulating the results for the decades of 1980s, 1990s, and 2000s.

Hedge portfolio alphas are fairly similar in January and in non-January months (0.76% and 0.74%, respectively), although not statistically significant for the former, likely due to small number of January observations. Our valuation model produces a hedge portfolio return of 1.73% in recessions and 0.86% in expansions, both are statistically significant. In unreported results, we find that the level of mispricing is a bit lower during the tech-boom

[12]Some readers have suggested that post-formation returns are not necessarily a sufficient test of the goodness of our valuation model, especially if market valuation drifts even further away from our 'fair' valuation. We check this by computing value gap, the difference between market valuation and our valuation. We calculate this value gap at portfolio formation and one quarter after portfolio formation (numbers not reported). We verify that the value gap does indeed shrink one quarter after portfolio formation. At the same time, the value gap does not decline to zero, suggesting that correction takes longer than one quarter (see also robustness checks on long horizon returns later in this section).

of the 1990s, although it shows some spikes around the last three U.S. recessions. Returns are also higher this century (1.03%) than in the previous two decades (0.70% and 0.91%).

We look at the horizon effects in Panel B of Table 4. Specifically, we consider holding periods of 3, 6, 12, and 18 months.[13] This implies that we have overlapping portfolios. We take equal-weighted average of these overlapping portfolios similar to the approach of Jegadeesh and Titman (1993). The table shows that 10−1 returns are strong and statistically significant for horizons up to 18 months, although they decline as we increase the horizon. In untabulated results, we look at month by month returns and find that most of the market value correction takes place in the first year after portfolio formation; there is almost no difference in returns between decile 1 and decile 10 in the 18th month after portfolio formation. We conclude that our valuation model does well, in general, across various subsamples and over longer horizons.

It is also interesting to examine how the information gets into the price. In other words, if the misvalued stocks converge to their fundamental values in the future, does the market learn about actual defaults or cash flows? To answer these questions, we perform three additional tests. First, as noted earlier in the previous section, we find that both under- and over-valued companies default more frequently than the rest of the stocks. Second, we calculate the option value as a fraction of total value at portfolio formation and one year after portfolio formation. We find that, for the extreme deciles one and ten, this fraction is 33% at portfolio formation but only 23% one year after. This suggests that the information about optionality is impounded in the price over the course of next year. Third, we follow the literature (eg. Lakonishok, Shleifer, and Vishny (1994) and Cooper, Gulen, and Schill (2008)) in examining what might be the triggers for this market learning. We replicate the results in Table 3 by partitioning the sample into firm-months with and without earnings announcements. The results show that with earnings announcement the model generates an alpha of 1.17% while the alpha is only 0.64% in months without earnings announcements; most of this difference arises from undervalued stocks. These tests show that the market learns slowly about both the actual defaults as well as cash flows of the misvalued firms.

[13]Whenever available from CRSP, we add delisting return to the last month traded return. If the delisting return is not available, we use the last full month return from CRSP. This could, in principle, impart an upward bias to portfolios returns (Shumway (1997)). However, CHS (2008) note, and we confirm, that this has no material impact on our results. Note that our procedure implies that the proceeds from sales of delisted stocks are reinvested in each portfolio in proportion to the weights of the remaining stocks in the portfolio.

3.2 The importance of default option in model valuation

Our valuation model is inspired by the option-like characteristics of common stocks. We claim that the option value can be a significant fraction of the total value of equity for some categories of stocks. In this section we verify the importance of this default option in the ability of the model to value stocks and thereby predict returns. We analyze the model's performance among subgroup of stocks for which the option to default is likely to be more relevant. We focus on four firm characteristics for this exercise: financial distress, size, market leverage, and volatility.

Perhaps, the most natural characteristic that one can associate with the relevance of the option to default is the extent of financial distress. In fact, the terms financial distress and high default risk are often used interchangeably: firms experiencing financial distress have more uncertainty about their ability to generate sufficient future cash flows, thus making the option to default particularly relevant for them. Put differently, for highly distressed firms the option to default is likely to be in-the-money, and thus captures significant fraction of the total equity value. We expect that our model's ability to detect misvaluation will be higher amongst financially distressed stocks. We employ the model of CHS (2008) to measure financial distress.[14] CHS use logit regressions to predict failure probabilities while incorporating a large set of accounting variables. Detailed description of the estimation procedure of this measure is provided in Appendix A.

The second characteristics that we consider is firm size. Since firm size is one input in the CHS distress measures, one can view firm size as a reduced-form proxy for the likelihood of default. Also, in general, young and small firms face more competitive challenges and higher capital constraints and are therefore more likely to default or abandon their business. We measure firm size by equity market value and expect that our model will perform better for small-cap stocks.

The third characteristic is financial leverage. While high leverage is not by itself a sign of financial distress, it is nevertheless true that there is a positive relation between the probability of default and leverage. We follow Penman, Richardson, and Tuna (2007) and calculate market leverage as the difference between financial liabilities and financial assets, divided by equity market value. Financial liabilities are equal to the sum of long-term debt, debt in current liabilities, carrying value of preferred stock, and preferred dividends in arrears, less preferred treasury stock. Financial assets are cash and short-term investments.

[14] Another common measure of distress is from the Moody's KMV model, which is based on the structural default model of Merton (1974), and largely relies on leverage ratio and asset volatility. In unreported results, we find very similar results using this measure as those using the CHS (2008) measure.

The market leverage for each month is given by the available accounting data at the beginning of the month (as in Penman, Richardson, and Tuna, we assume a four-month lag of annual statements) and the equity value as of the end of the last month. Our model is expected to deliver better performance for highly levered stocks.

The fourth and last firm characteristic is stock return volatility. The high uncertainty about the future of firms facing the possibility of default is likely to be reflected in high stock return volatility. In particular, any news about future cash flows that affects the likelihood that the firm will default has a strong impact on the current price. In turn, as implied by option pricing theory, the value of option increases with the volatility of the underlying asset. We follow Ang, Hodrick, Xing, and Zhang (2006) and calculate idiosyncratic volatility for each month by the standard deviation of the residuals of regression of daily stock returns on the daily Fama-French (1993) three factors augmented with the momentum factor. For each month, the idiosyncratic volatility is estimated during the previous month.[15] We expect better model performance for highly volatile stocks.

We proceed as follows to examine the effect of these characteristics on the returns to the relative valuation portfolios. Each month we first sort all stocks into five quintiles according to each characteristic, using current market data and quarterly accounting data of the previous quarter. Then, within each characteristic quintile, we sort all stocks into five equal-sized portfolios according to the model value to market value. These second-sorted portfolios are labeled R1 (most overvalued) to R5 (most undervalued). Our double-sorted portfolios are well populated as the average number of stocks per portfolio is 123.

We report the mean fraction of default option value in the total equity value implied by our valuation model in the last column of each characteristic panel of Table 5. To compute this fraction we run the model while shutting down the default option by disallowing default and exit and forcing equityholders to operate the firm indefinitely. The value of the option to default is then given by the difference in equity values with and without this option (see equation (4)). These mean fractions confirm our choice of characteristics, as they increase with distress, leverage, and volatility, and decrease with firm size. For example, option value is, on average, 42.5% of the total value of equity for the most distressed stocks, but only 21.3% of the total value of equity for the least distressed stocks. Note that our model captures both the default and abandonment options and, therefore, implies positive option values for even zero-leveraged firms as long as fixed costs are nonzero. Mean fraction is increasing from 19.7% to 33.2% when one moves from low- to high-leverage stocks, and increasing from 19.3% to 35.7% when one moves from low- to high-volatility stocks. Size has a weaker effect

[15] The results remain similar taking the average idiosyncratic volatility during the prior three months and during the prior twelve months, or using total volatility instead of idiosyncratic volatility.

on default option; mean fractions of 21.3% and 31.9% for the large and small size quintiles, respectively.

Table 5 also shows the 4-factor alphas on all these double-sorted portfolios. For each characteristic sort we find that (a) the 4-factor alphas for R5 (most undervalued stocks) are always higher than those for R1 (most overvalued stocks), and (b) the hedge portfolio alphas are increasing with the value of default option. For example, Panel A shows that hedge portfolio alpha is only 0.23% (t-statistic=1.25) for least distressed stocks quintile and increases monotonically to 1.32% (t-statistic=2.38) for the top distressed quintile. The effect of size in Panel B on the returns generated by the model's relative values portfolios is somewhat weaker than those of distress; the 4-factor alpha of the hedge portfolio is 0.66% for the small size quintile and 0.48% for the large stock quintile. These return patterns are consistent with the weaker relation between size and the fraction of default option shown in the last column of Panel B. Panel C shows no evidence that our model does better for highly levered stocks; the 4-factor alpha of the hedge portfolio is 0.80% for the low-leverage stocks but smaller at 0.63% for the high-leverage stocks. Panel D shows that the 4-factor alpha of the hedge portfolio is 1.25% for the high-volatility stocks (for which default option is more valuable), whereas it is only 0.50% for the low-volatility stocks (where default option is less valuable).

The returns in Table 5, thus, show that the strength of the model in valuing stocks is largely driven by the option to default. To provide a more direct test of the importance of default option in the total equity value, we also recompute the returns to our double-sorted portfolios by shutting down the default option. Table 6 shows the 4-factor alphas to the long-short R5−R1 portfolios for each characteristic quintiles. The performance of the model in predicting returns deteriorates sharply without the option to default. For the top quintile of distressed stocks and idiosyncratic volatility stocks, the 4-factor alpha of the model without the default option is roughly half the magnitude of the model with the default option. For example, there is reduction in 4-factor alpha from 1.32% to 0.57% for most distressed stocks, and from 1.25% to 0.43% for most volatile stocks. In contrast, reduction is alpha is relatively modest for small stocks and, contrary to our expectations, increases for highly levered stocks.

It is important to note a limitation of our results. While our valuation model performs best amongst the subset of securities with most valuable default options, the average returns across R1 to R5 quintiles among those stocks are often negative and also lower than the equivalent returns for the stock with the least valuable default options. For example, the average 4-factor alpha across the R1 to R5 quintiles is −0.31% for the top distress D5 quintile and 0.20% for the bottom distress D1 quintile. These relatively low returns to distressed stocks are refereed to as the distress puzzle (CHS (2008)) in the literature. This fact implies that our valuation model still misses common factors relevant to stock values. We, therefore, urge caution in using our model as an absolute valuation model and prefer to use it as a means of relative valuation. We also reiterate that the power of our model is in the valuation of the option to default. For stocks far from the boundary of default, normal valuation techniques could still be adequate and not much may be gained by using our model for such stocks.

3.3 Fama-MacBeth regressions on relative model value

The portfolio sorts provide a simple view of the relation between returns and our variables of interest. Another approach commonly used in the literature is that of Fama and MacBeth (1973) regressions. Beyond serving as an additional diagnostic check, these regressions offer the advantage that we can control for other well-known determinants of the cross-sectional patterns in returns and thus check for the marginal influence of relative model valuation on our results. Accordingly, we run these cross-sectional regressions and report the results in Table 7. The dependent variable is the excess stock return while the independent variables are (log) market capitalization, (log) market-to-book, past six-month return, relative model value (log of the ratio of the equity value implied by our valuation model to the actual equity value; higher numbers indicate undervaluation based on our model), CHS distress risk measure, market leverage, volatility, and interaction terms between relative model value and the characteristics of interest.[16] We winsorize all independent variables at the 1% and 99% levels to reduce the impact of outliers. All reported coefficients are multiplied by 100 and we report Newey-West (1987) corrected (with six lags) t-statistics in parentheses.

The first regression shows the usual patterns; returns are related to size, market-to-book, and past return. Specifications (3) shows that relative model value is positively associated with future returns. While this effect was present in the portfolio sorts shown in Table 3, the regression shows that the ability of our model to value stocks survives the inclusion of other stock characteristics. Specification (4) shows that the interaction terms between relative model value and distress is positive and significant with a t-statistics of 3.78. This implies that our relative model value does particularly well for the subset of distressed stocks. The effect of the interaction term on size in specification (5) is highly statistically significant in contrast to the weaker results from portfolio sorts. However, regressions in specification (6) continue to show no improvement in model performance for highly levered stocks. Finally,

[16]We do not control for some other additional characteristics such as asset growth (Cooper, Gulen, and Schill (2008) and equity issuance (Pontiff and Woodgate (2008)). These characteristics are correlated with our relative valuation measures. For instance, as reported earlier, we view the fact that overvalued stocks issue more equity as a vindication of our model rather than treat equity issuance as competitor variable to relative model valuation in explaining future returns.

specification (7) show that the performance of the model significantly improves when applied to highly volatile stocks (t-statistic of the interaction terms is 1.95). These regression results, coupled with the portfolio sort results provided in Tables 5 and 6, thus, demonstrate the importance of the default option in the model valuation.

4 Anomalies

We next turn our attention to the analysis of stock return anomalies that might be associated with the option to default. The evidence in the previous section indicates that our option-based valuation model detects stocks whose market values deviate from their fundamental values. We further show in Table 2 that stocks misvalued according to our model have characteristics that are typical to stocks that are most difficult to value; i.e. they are relatively small, less liquid, have fewer analyst coverage with high earnings forecast dispersion, and have lower institutional ownership. We, therefore, conjecture that the apparent difficulty to incorporate appropriately the default option in stock valuation creates a fruitful ground for return anomalies related to default option to persist. While we are silent on the exact mechanisms driving various anomalies, we argue that anomalies are more widespread in most misvalued stocks according to our model.

The three anomalies that we analyze are based on the three characteristics as in Section 3.2 viz. distress (Dichev (1998) and Campbell et al. (2008)), leverage (Penman, Richardson, and Tuna (2007)), and idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang (2006)).[17] An important question behind all three anomalies is whether they are evidence of market inefficiency (mispricing) or the result of rational pricing that we do not yet understand. For example, CHS (2008) view the distress anomaly as mispricing, but Gomes and Schmid (2010) and George and Hwang (2009) argue that low returns to distress stocks have a rational basis. Our objective is to contribute to this debate by estimating the extent to which returns to these anomalies are related to the mispricing identified by the model, and are therefore likely attributable to mispricing of default option.

4.1 Returns of portfolio sorted on value gap and stock characteristics

Since our conjecture is more directly related to misvaluation rather than under- or overvaluation, we classify stocks as fairly valued and misvalued, in contrast to the previous section

[17]We thank the referee and the editor for suggesting these anomalies.

where we considered under- and overvalued stocks separately. In particular, we double-sort all stocks based on two variables. The first sorting variable is value gap, defined as the absolute value of the log of the ratio of the equity value implied by our valuation model and the actual equity value. We label these portfolios VG1 to VG3; VG1 has the least value gap (most fairly valued stocks) while VG3 has the highest value gap (most misvalued stocks). Within each value gap tercile we sort all stocks into quintiles based a stock characteristic representing each anomaly. As before, the double-sorting is done at the end of each month and we hold these portfolios for one month. All portfolio sort results are robust to different subsamples and horizons (not reported).

Table 8 reports the 4-factor alphas to these double-sorted portfolios. The portfolios of the most interest to us are the long-short portfolios sorted on firm characteristic for each value-gap tercile (the last row in each panel). Panel A considers financial distress as the stock characteristic sorting variable. The hedge portfolio (long least distressed stocks, short most distressed firms) alpha increases monotonically from 0.56% for VG1 tercile to 0.82% for VG3 tercile. It is worth noting that the higher hedge portfolio alpha in VG3 tercile relative to VG1 tercile is due to both overperformance of least distressed stocks and underperformance of most distressed stocks. The evidence that the distress anomaly is more apparent among most misvalued stocks according to our model is consistent with our argument that option-like features of common stocks present valuation difficulties to investors. Panel B does not show a clear relation between the leverage anomaly and misvaluation. The leverage effect is the strongest for the middle value gap stocks, and the weakest among most misvalued stocks. However, the results for the idiosyncratic volatility anomaly in Panel C support our conjecture. The low-high idiosyncratic volatility portfolio alpha increases monotonically from 0.25% for VG1 tercile to 0.81% for VG3 tercile. The effect of misvaluation on the idiosyncratic volatility anomaly is driven mostly by underperformance of high-volatility stocks.

Therefore, our results indicate that distress and idiosyncratic volatility anomalies are related to stock mispricing, especially due to the default option value. However, we do not find evidence for the possibility that the leverage anomaly is related to mispricing, as identified by our model.

4.2 Fama-MacBeth regressions on value gap

We check the strength of our results via cross-sectional Fama-MacBeth regressions similar to those in Section 3.3. The independent variables include, as before, (log) market capitalization, (log) market-to-book, past six-month return, as well as value gap, CHS distress-risk measure, market leverage, idiosyncratic volatility, and interaction terms. The results are reported in Table 9.

In specifications (1), (3), and (5) we verify the existence of the three anomalies in our sample. The coefficients of CHS, market leverage, and idiosyncratic volatility are all negative and significant (t-statistics of -6.71, -5.37, and -3.37), showing that highly distressed, highly levered, and highly volatile stocks earn significantly lower subsequent returns. More importantly, the coefficients of the interaction terms corroborate the portfolio sort returns. Specifications (2) and (6) show, respectively, that the distress anomaly and the idiosyncratic volatility anomaly are stronger in firms with higher value gap (t-statistics of the interaction variables are -3.49 and -2.23). Specification (4) shows, consistent with portfolio sorts results, a positive, albeit statistically insignificant, effect of value gap on the leverage anomaly.

Both portfolio sorts and Fama-MacBeth regressions, therefore, show that the puzzling low returns to distressed stocks and to stocks with high idiosyncratic volatility are much more pronounced amongst stocks that are misvalued according to our model. Because our model's strength in detecting misvaluation relies on the option to default, we conjecture that the inability of investors to value default option correctly contributes significantly to the prevalence of these anomalies.

To verify this conjecture more directly we replicate the Fama-MacBeth regressions in Panel A while shutting down the option to default in model valuation. The coefficients of the interaction terms reported in Panel B show that most of the effect of the extent of misvaluation on the existence of the anomalies is reduced without the option to default. The t-statistic of CHS/value gap interaction variable is reduced from -3.49 to -1.42, and the t-statistic of idiosyncratic volatility/value gap interaction is reduced from -2.23 to -0.53. These results support our conjecture that misvaluation of default option is a key ingredient in generating these anomalies. The coefficient on the interaction term for leverage is statistically significant, but the wrong sign from our expectations.

Finally, we recognize that the extent of misvaluation according to our model can be correlated with valuation uncertainty according to other sources. A natural place to look for the level of valuation uncertainty is analysts' reports; higher dispersion in earnings' forecasts indicate higher level of valuation uncertainty. As reported in Table 2, stocks that are more misvalued according to our model exhibit higher standard deviation of analysts' forecasts. To confirm that our model detects a source of misvaluation that is not picked up by analysts' reports, we replicate the regressions in Table 9 controlling for the effect of analyst valuation uncertainty on the extent of the anomalies. We control for the standard deviation of analysts' forecasts as well as the number of analysts. Both these variables are also interacted with distress, leverage, and idiosyncratic volatility. In unreported regression results, we find that the effect of misvaluation according to our model on the distress and idiosyncratic volatility anomalies remains significant in the presence of these additional variables.

5 Conclusion

Equities are embedded with an option to default. We believe that a meaningful equity valuation model should take this optionality into account. We build such a model by accounting for the value of the option to default. Our model does a good job in separating over- and undervalued stocks. The long-short strategy that buys stocks that are classified as undervalued by our model and shorts overvalued stocks generates an annualized 4-factor alpha of about 11%. This performance is robust to various sample splits and holding periods. Furthermore, a similar investment strategy produces significantly higher returns for stocks with relatively high value of default option, namely distressed, highly volatile, and small stocks, articulating the importance of the option to default as the key ingredient of our model. This suggests that, in general, investors do not recognize the option-like nature of equities and do not value them accordingly.

We also argue that investors' difficulties in valuing the option to default is related to the existence of stock return anomalies, especially those associated with default option. In particular, stocks with high default option value are more volatile and less liquid, and have low analyst coverage with high dispersion of earnings forecasts. These valuation difficulties create valuation uncertainty and provide a fruitful ground for return anomalies to persist. To support this view, we deploy our valuation model to further classify stocks into fairly priced versus mispriced based on their relative valuation. We find that distress and idiosyncratic volatility anomalies are concentrated among most misvalued stocks. This supports our conjecture that anomalies are driven by investors' inability to value the option to default correctly.

Appendix A: Distress Measure

Campbell, Hilscher, and Szilagyi (2008) use logit regressions to predict failure probabilities. We use their model for predicting bankruptcy over the next year (model with lag 12 in their Table IV) as our baseline model. This model, which is repeated below, gives the probability of bankruptcy/failure from a logit model as:

$$\begin{aligned} CHS_t &= -9.16 - 20.26\, NIMTAAVG_t + 1.42\, TLMTA_t - 7.13\, EXRETAVG_t \\ &\quad +1.41\, SIGMA_t - 0.045\, RSIZE_t - 2.13\, CASHMTA_t + 0.075\, MB_t \\ &\quad -0.058\, PRICE_t, \end{aligned} \tag{A1}$$

where $NIMTA$ is the net income divided by the market value of total assets (the sum of market value of equity and book value of total liabilities), $TLMTA$ is the book value of total liabilities divided by market value of total assets, $EXRET$ is the log of the ratio of the gross returns on the firm's stock and on the S&P500 index, $SIGMA$ is the standard deviation of the firm's daily stock return over the past three months, $RSIZE$ is ratio of the log of firm's equity market capitalization to that of the S&P500 index, $CASHMTA$ is the ratio of the firm's cash and short-term investments to the market value of total assets, MB is the market-to-book ratio of the firm's equity, and $PRICE$ is the log price per share. $NIMTAAVG$ and $EXRETAVG$ are moving averages of $NIMTA$ and $EXRET$, respectively, constructed as (with $\phi = 2^{-\frac{1}{3}}$):

$$\begin{aligned} NIMTAAVG_{t-1,t-12} &= \frac{1-\phi^3}{1-\phi^{12}} \left(NIMTA_{t-1,t-3} + \ldots + \phi^9 NIMTA_{t-10,t-12} \right), \\ EXRETAVG_{t-1,t-12} &= \frac{1-\phi}{1-\phi^{12}} \left(EXRET_{t-1} + \ldots + \phi^{11} EXRET_{t-12} \right). \end{aligned} \tag{A2}$$

The source of accounting data is COMPUSTAT while all market level data are from CRSP. All accounting data are taken with a lag of three months for quarterly data and a lag of six months for annual data. All market data used in calculating the distress measure of equation (A1) are the most current data. We winsorize all inputs at the 5th and 95th percentiles of their pooled distributions across all firm-months (winsorizing at the 2nd and 98th percentiles has no material impact on our results), and $PRICE$ is truncated above at \$15. Further details on the data construction are provided by CHS (2008) and we refer the interested reader to their paper.[18] We include all common stocks, although our results are robust to the exclusion of financial stocks. The sample period for our study is 1983 to 2012 as the coverage of quarterly COMPUSTAT data is sparse before this date.

[18] There are two minor differences between CHS's (2008) approach and ours. First, CHS eliminate stocks with fewer than five nonzero daily observations during the last three months; and then replace missing $SIGMA$ observations with the cross-sectional mean $SIGMA$ in estimating their bankruptcy prediction regressions. We do not make this adjustment. Second, CHS treat firms that fail as equivalent to delisted firms, even if CRSP continues to report returns for these firms. We do not make this adjustment either.

Appendix B: Numerical Details on the Valuation Model

The first step is to find a value of the firm that survives until year five and pays off its long-term debt. We assume that at the end of year five, the firm refinances by issuing perpetual coupon debt in an amount to match the average SIC3 market leverage ratio. We assume refinancing to average industry leverage, as opposed to inferring the optimal leverage from the model due to the known tendency of structural contingent claim models to predict optimal leverage ratios that appear too high compared with their empirical counterparts.

The net instantaneous post-refinancing cash flow to equityholders is:

$$CF_{it} = \left[(x_{it} - c_i - F_i)(1-\tau) + \tau Dep_{it} - Capex_{it} + \eta \mathbf{1}_{(x_{it}-c_i-F_i)(1-\tau)+\tau Dep_{it} - Capex_{it}<0}\right] dt, \tag{B1}$$

where the coupon amount is c_i. The cash flow to bondholders is $c_i dt$. Note that the additional cost of financial distress η is incurred if $x_{it} < x^*$, where:

$$(x^* - c_i - F_i)(1-\tau) + \tau Dep_{it} - Capex_{it} = 0\,.$$

Because we assume that the gross margin GM_{it} as well as depreciation-to-sales and capex-to-sales ratio stay constant over time, x^* is given by:

$$x^* = \frac{(c_i + F_i)(1-\tau)}{1 - \tau + \left(\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}\right)/GM_{it}}\,. \tag{B2}$$

The cash flows to equityholders and, therefore, the value of equity depend on whether the current value of x_{it} is above or below the threshold x^*. The cash flows in equation (B1) above can be rewritten as:

$$\begin{aligned} CF_{it} \;=\; & \Bigg[x_{it}\left(\frac{\tau\overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right) \\ & - (c_i + F_i)(1-\tau) + \eta \mathbf{1}_{x_{it}\left(\frac{\tau\overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right) - (c_i+F_i)(1-\tau)<0}\Bigg] dt. \end{aligned}$$

Then standard arguments show that the value of equity is given by:

$$E(x_{it}) = \begin{cases} Ax_{it}^{\beta_1} + Bx_{it}^{\beta_2} + \left[\frac{\tau\overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right]\frac{x_{it}}{r-\mu} - (1-\tau)\frac{c_i+F_i}{r} & \text{if } x_{it} > x^* \\ Cx_{it}^{\beta_1} + Dx_{it}^{\beta_2} + (1+\eta)\left\{\left[\frac{\tau\overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right]\frac{x_{it}}{r-\mu} - (1-\tau)\frac{c_i+F_i}{r}\right\} & \text{if } x_{it} < x^*, \end{cases} \tag{B3}$$

where β_1 and β_2 are the positive and the negative root of the quadratic equation $\frac{1}{2}\sigma^2\beta(\beta - 1) + \mu_Q\beta - r = 0$, and A, B, C, and D are constants. Equation (B3) must be solved subject to the following boundary conditions:

$$
\begin{aligned}
&A = 0 \\
&Bx^{*\beta_2} + \left[\frac{\tau\overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right]\frac{x^*}{r-\mu_Q} - (1-\tau)\frac{c_i+F_i}{r} = \\
&\qquad Cx^{*\beta_1} + Dx^{*\beta_2} + (1+\eta)\left\{\left[\frac{\tau\overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right]\frac{x^*}{r-\mu_Q} - (1-\tau)\frac{c_i+F_i}{r}\right\} \\
&\beta_2 Bx^{*\beta_2-1} + \left[\frac{\tau\overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right]\frac{1}{r-\mu_Q} = \\
&\qquad \beta_1 Cx^{*\beta_1-1} + \beta_2 Dx^{*\beta_2-1} + (1+\eta)\left\{\frac{\tau\overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right\}\frac{1}{r-\mu_Q} \\
&Cx_d^{\beta_1} + Dx_d^{\beta_2} + (1+\eta)\left\{\left[\frac{\tau\overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right]\frac{x_d}{r-\mu_Q} - (1-\tau)\frac{c_i+F_i}{r}\right\} = 0 \\
&\beta_1 Cx_d^{\beta_1-1} + \beta_2 Dx_d^{\beta_2-1} + (1+\eta)\left[\frac{\tau\overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right]\frac{1}{r-\mu_Q} = 0.
\end{aligned}
\tag{B4}
$$

The first boundary condition precludes bubbles as x increases, the second and third conditions ensure that the value functions and their first derivatives match at x^*, and the fourth and fifth conditions are the value-matching and smooth-pasting conditions that ensure optimality of the default threshold x_d. Together, these conditions comprise a system of four non-linear equations with four unknowns (B, C, D, and x_d) that must be solved numerically. By solving this system we find the post-refinancing value of equity in year five, $E(x_{i5})$.

The value of debt is given by:

$$
D(x_{i5}) = \frac{c_i}{r} + \left(\frac{x_{i5}}{x_d}\right)^{\beta_2}\left[(1-\alpha)\max(V_U(x_d), 0) - \frac{c_i}{r}\right], \tag{B5}
$$

where $V_U(x_d)$ is the value of the unlevered firm and α is the bankruptcy costs (upon default debtholders get this unlevered value, if positive, net of bankruptcy costs). When implementing this procedure, we set $\alpha = \eta = 15\%$.

For a given x_{i5} we find the value of c_i such that $\frac{D(x_{i5})}{E(x_{i5})+D(x_{i5})}$ is equal to the average SIC3 leverage ratio in the last three years. If we are unable to find this solution (e.g. for high enough values of fixed costs), we assume that the firm remains unlevered throughout the rest of its life. The pre-refinancing equity value is :

$$
E'(x_{i5}) = E(x_{i5}) + D(x_{i5}). \tag{B6}
$$

Once we find the terminal value of equity in year five, $E'(x_{i5})$, we solve the model numerically and compute the optimal default boundary and equity values for all $t \leq T = 5$. For that purpose, we introduce a new variable $y_t = \log(x_t)$, that follows an arithmetic Brownian motion under the risk-neutral measure:

$$dy_t = \left(\mu_Q - \frac{\sigma^2}{2}\right) dt + \sigma dW_t. \tag{B7}$$

We then discretize the problem by using a two-dimensional grid $N_y \times N_t$ with the corresponding increments of y and t given by dy and dt, where $dy = (y_{\max} - y_{\min})/N_y$ and $dt = T/N_t$, where $T = 5$. To get a reasonable balance between execution speed and accuracy we set $dt = 0.1$, $y_{\min} = -5$, and $y_{\max} = 10$ when implementing this algorithm.

We iterate valuations backwards using a binomial approximation of the Brownian motion (see, for example, Dixit and Pindyck (1994)). At each node the equityholders have an option to default. They will default if the present value (under Q) of running the firm for one more period is negative:

$$\begin{aligned} E(ndy, mdt) = \max\Big\{&(1 - rdt)\left[p_u E((n+1)dy, (m+1)dt) + p_d E((n-1)dy, (m+1)dt\right] \\ &+ \Bigg[e^{n\times dy}\left(\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right) - (I_{it} + F_i)(1-\tau) - D_{it} \\ &+ \eta \mathbf{1}_{x_{it}\left(\frac{\tau \overline{DSR}_{t-3,t} - \overline{CSR}_{t-3,t}}{GM_{it}} + (1-\tau)\right) - (I_i + F_i)(1-\tau) < 0}\Bigg] dt, 0\Big\}, \end{aligned} \tag{B8}$$

where

$$p_u = 0.5 + \left(\mu_Q - \frac{\sigma^2}{2}\right)\frac{\sqrt{dt}}{2\sigma}, \quad p_d = 1 - p_u, \text{ and } dy = \sigma\sqrt{dt}.$$

Equation (B8) shows that at each node the value of equity is given by the discounted present value of equity the next time period plus the cash flows that equityholders receive over the time period dt. If this value is negative, then the firm is below the optimal default boundary so it is optimal for equityholders to default, in which case the value of equity is zero. (We assume that the absolute priority rule is enforced if bankruptcy occurs and the residual payout to equityholders is zero.)

References

Amihud, Yakov, 2002, "Illiquidity and Stock Returns: Cross-Section and Time-Series Effects," *Journal of Financial Markets* 5, 31–56.

Andrade, Gregor, and Steven Kaplan, 1998, "How Costly is Financial (not Economic) Distress? Evidence from Highly Leveraged Transactions that Become Distressed," *Journal of Finance* 53, 1443–1493.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006, "The Cross-Section of Volatility and Expected Returns," *Journal of Finance* 61, 259–299.

Benartzi, Shlomo, and Richard H. Thaler, 2001, "Naïve Diversification Strategies in Defined Contribution Saving Plans," *American Economic Review* 91, 79–98.

Black, Fischer, and Myron Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, 637–654.

Brealey, Richard, Stewart Myers, and Franklin Allen, 2011, *Principles of Corporate Finance*, McGraw-Hill/Irwin.

Campbell, John Y., Jens Hilscher, and Jan Szilagyi, 2008, "In Search of Distress Risk," *Journal of Finance* 63, 2899–2939.

Cooper, Michael J., Huseyin Gulen, and Michael J. Schill, 2008, "Asset Growth and the Cross-Section of Stock Returns," *Journal of Finance* 63, 1609–1651.

Delianedis, Gordon, and Robert Geske, 2003, "Credit Risk and Risk Neutral Default Probabilities: Information about Rating Migrations and Defaults," Working paper, Anderson School UCLA.

Dichev, Ilia, 1998, "Is the Risk of Bankruptcy a Systematic Risk?" *Journal of Finance* 53, 1141–1148.

Dixit, Avinash K., and Robert S. Pindyck, 1994, *Investment Under Uncertainty*, Princeton University Press, Princeton.

Elkamhi, Redouane, Jan Ericsson, and Christopher A. Parsons, 2011, "The Cost of Financial Distress and the Timing of Default," *Journal of Financial Economics* 47, 427–465.

Fama, Eugene F., and Kenneth R. French, 1993, "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics* 33, 3–56.

Fama, Eugene F., and James D. MacBeth, 1973, "Risk, Return and Equilibrium: Empirical Tests," *Journal of Political Economy* 81, 607–636.

Garlappi, Lorenzo, and Hong Yan, "Financial Distress and the Cross-Section of Equity Returns," *Journal of Finance* 66, 789–822.

George, Thomas J., and Chuan-Yang Hwang, 2009, "A Resolution of the Distress Risk and Leverage Puzzles in the Cross Section of Stock Returns," *Journal of Financial Economics* 96, 56–79.

Geske, Robet, 1978, "The Valuation of Compound Options," *Journal of Financial Economics* 7, 63–81.

Goldstein, Robert, Nengjiu Ju, and Hayne E. Leland, 2001, "An EBIT-Based Model of Dynamic Capital Structure," *Journal of Business* 74, 483–512.

Gomes, J. F., and L. Schmid, 2010, "Levered Returns," *Journal of Finance* 65, 467–494.

Hirshleifer, David, and Siew H. Teoh, 2003, "Limited Attention, Information Disclosure, and Financial Reporting," *Journal of Accounting and Economics* 36, 337–386.

Hwang, Lee-Seok, and Byungcherl Charlie Sohn, 2010, "Return Predictability and Shareholders' Real Options," *Review of Accounting Studies* 15, 367–402.

Jegadeesh, Narasimhan, and Sheridan Titman, 1993, "Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency," *Journal of Finance* 48, 65–91.

Lakonishok, Josef, Andrei Shleifer, and Robert Vishny, 1994, "Contrarian Investment, Extrapolation, and Risk," *Journal of Finance* 49, 1541–1578.

Lee, Charles M.C., James Myers, and Bhaskaran Swaminathan, 1999, "What is the Intrinsic Value of the Dow?" *Journal of Finance* 54, 1693–1741.

Leland, Hayne E., 1994, "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance* 49, 1213–1252.

Lewellen, Jonathan W., 2004, "Predicting Returns with Financial Ratios," *Journal of Financial Economics* 74, 209–235.

Merton, Robert C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance* 29, 449–470.

Moon, Mark, and Eduardo S. Schwartz, 2000, "Rational Pricing of Internet Companies Revisited," *Financial Analyst Journal* 56, 62–75.

Newey, Whitney K., and Kenneth D. West, 1987, "A Simple Positive Semidefinite Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica* 55, 703–708.

Ozdagli, Ali K., 2010, "The Distress Premium Puzzle," FRB Boston Working Papers Series 10-13.

Pástor, Ľuboš, and Robert F. Stambaugh, 2003, "Liquidity Risk and Expected Stock Returns," *Journal of Political Economy* 111, 642–685.

Pontiff, Jeffrey, and Artemiza Woodgate, 2008, "Share Issuance and Cross-Sectional Returns," *Journal of Finance* 63, 921–945.

Penman, Stephen H., Scott Richardson, and A. Irem Tuna, 2007, "The Book-to-Price Effect in Stock Returns: Accounting for Leverage," *Journal of Accounting Research* 45, 427–467.

Poteshman, Allen, and Vitaly Serbin, 2003, "Clearly Irrational Market Behavior: Evidence from the Early Exercise of Exchange Traded Options," *Journal of Finance* 58, 37–70.

Shumway, Tyler, 1997, "The Delisting Bias in CRSP Data," *Journal of Finance* 52, 327–340.

Weiss, Lawrence A., 1990, "Bankruptcy Resolution: Direct Costs and Violation of Priority of Claims," *Journal of Financial Economics* 27, 285–314.

Table 1: Inputs to the Valuation Model

This table reports the input parameters used in our valuation model for all CRSP/COMPUSTAT firm population. The categories of parameters include values that are kept constant for all firms and months, firm-month specific values, and values based on 2-digit SIC industry code and CHS (2008) distress-risk quintile. The sample period is 1983 to 2012.

Input variable	Value used in the model	Mean	Median	StDev
Coupon rate	AAA, BBB and BBB+2% yields for distress quintiles 1-2, 3-4, and 5, respectively	8.35%	8.04%	2.36%
Distress costs, η	15%			
Corporate tax rate, τ	35%			
Risk-free rate, r	Avg. of 3-month and 10-year treasury yields	5.22%	5.25%	2.56%
R_{WACC}	Avg. industry-distress WACC in the last three years	9.39%	9.56%	2.49%
CAPEX to sales ratio, CSR	Avg. industry-distress CSR in the last three years	0.108	0.066	0.122
Depreciation to sales ratio, DSR	Avg. industry-distress DSR in the last three years	0.079	0.048	0.079
Volatility, σ (annualized)	Quarterly volatility of sales	0.396	0.260	0.440
Short term debt/Total assets	Annual COMPUSTAT items DLC/AT	0.057	0.020	0.114
Long term debt/Total assets	Annual COMPUSTAT items DLTT/AT	0.169	0.110	0.203
Market leverage ratio	(DLC+DLTT)/(DLC+DLTT+Equity value)	0.265	0.191	0.258
Fixed costs/Sales	Annual COMPUSTAT items XSGA/SALE	0.351	0.244	0.466
Gross margin/Sales	Annual COMPUSTAT items (SALE–COGS)/SALE	0.253	0.346	0.874

Table 2: Characteristics of Portfolios Sorted on Relative Model Value

Each month, we sort all stocks into deciles according to the ratio of the equity value implied by our valuation model to the actual equity value (Decile 1=most overvalued, Decile 10=most undervalued). The portfolios are value-weighted and held for one subsequent month. The table presents descriptive statistics for each portfolio, where for all variables, observations outside the top and the bottom percentiles are excluded. For each characteristic, we first calculate the cross-sectional mean and median across stocks for each portfolio. The table then reports the time-series averages of these means/medians. Size is equity value (in millions of dollars). Market-to-book ratio is equity market value divided by equity book value. Market beta is measured by regression of stock return on market return over the past 60 months. Past return is cumulative return over the past six months. Standard deviation of daily stock returns (reported in percent) is based on market-adjusted returns in the past year. Share turnover is trading volume scaled by total shares outstanding. Amihud illiquidity is the monthly average of daily ratios of absolute return to dollar trading volume (in millions). Percent of firms with negative earnings is based on the net income in the previous calendar year. Number of analysts covering the firm is measured by the number of forecasts appearing in IBES. Standard deviation of analysts' forecasts is also calculated from IBES data. Equity issuance (reported in percent) is measured by the difference between the sale and purchase of common and preferred stocks during the year, scaled by equity market value at the beginning of the year. Institutional ownership (reported in percent) is the sum of all shares held by institutions divided by total shares outstanding. The sample period is 1983 to 2012.

		1	2	3	4	5	6	7	8	9	10
Size	Mean	833.4	1,757.0	2,197.9	2,057.0	2,034.1	1,822.5	1,605.7	1,313.5	1,037.9	435.7
	Median	113.6	260.1	384.1	390.0	357.8	308.9	275.4	220.3	156.8	55.4
Market-to-book ratio	Mean	2.69	2.57	2.40	2.16	1.96	1.79	1.64	1.51	1.34	1.04
	Median	2.11	2.24	2.18	1.96	1.76	1.60	1.46	1.32	1.15	0.80
Market beta	Mean	1.31	1.28	1.15	1.04	0.98	0.94	0.93	0.95	0.95	1.01
	Median	1.19	1.17	1.05	0.93	0.88	0.84	0.83	0.86	0.87	0.94
Past return	Mean	15.9	17.7	15.3	12.6	10.3	8.3	6.6	4.4	1.1	-7.2
	Median	6.6	9.0	9.0	7.7	5.9	4.5	2.7	0.2	-3.3	-12.2
Stdev of stock returns	Mean	4.2	3.6	3.2	3.0	2.9	2.9	3.0	3.2	3.7	4.9
	Median	3.7	3.1	2.7	2.5	2.4	2.4	2.5	2.7	3.1	4.2
Share turnover	Mean	0.12	0.13	0.12	0.11	0.10	0.09	0.09	0.09	0.08	0.08
	Median	0.07	0.08	0.08	0.07	0.07	0.06	0.06	0.06	0.06	0.05
Amihud's illiquidity	Mean	5.15	3.28	2.54	2.37	2.49	2.73	3.28	4.55	6.95	14.83
	Median	0.21	0.07	0.05	0.06	0.06	0.07	0.11	0.26	0.63	2.19
% of negative earnings	Mean	60.9	33.6	23.0	17.7	15.7	15.1	15.6	17.8	22.0	35.9
	Median	86.0	10.5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.5
Number of analysts	Mean	3.54	4.01	4.22	4.11	3.87	3.79	3.63	3.30	3.19	2.60
	Median	2.80	3.27	3.51	3.47	3.25	3.14	3.00	2.71	2.65	2.15
Stdev of analysts' forecasts	Mean	0.06	0.04	0.04	0.04	0.04	0.05	0.06	0.06	0.06	0.08
	Median	0.04	0.03	0.03	0.03	0.03	0.04	0.05	0.04	0.05	0.06
Equity issuance	Mean	5.9	3.1	1.9	1.3	1.0	1.0	1.0	1.0	1.3	3.0
	Median	0.7	0.2	0.2	0.1	0.0	0.1	0.1	0.0	0.0	0.0
Institutional ownership	Mean	31.5	40.7	44.4	44.3	43.0	41.1	40.0	38.6	35.2	29.5
	Median	25.7	38.9	45.6	45.8	43.6	40.9	38.9	37.4	33.1	25.5

Table 3: Returns of Portfolios Sorted on Relative Model Value

Each month, we sort all stocks into deciles according to the ratio of the equity value implied by our valuation model to the actual equity value (Decile 1=most overvalued, Decile 10=most undervalued). The portfolios are value-weighted and held for one subsequent month. The table shows the portfolios' mean excess monthly returns (in excess of the risk-free rate) and alphas from factor models. The CAPM one-factor model uses the market factor. The three factors in the 3-factor model are the Fama and French (1993) factors. The four factors in the 4-factor model are the Fama-French factors augmented with a momentum factor. All returns and alphas are in percent per month and the corresponding t-statistics are in parentheses. The sample period is 1983 to 2012.

	1	2	3	4	5	6	7	8	9	10	10−1
Excess return	0.51	0.42	0.67	0.67	0.54	0.73	0.77	0.86	1.12	1.13	0.63
	(1.37)	(1.38)	(2.47)	(2.67)	(2.22)	(2.94)	(3.10)	(3.40)	(4.04)	(3.24)	(2.05)
CAPM alpha	-0.31	-0.28	0.06	0.10	0.00	0.19	0.25	0.33	0.55	0.48	0.79
	(-1.79)	(-2.28)	(0.52)	(1.01)	(0.04)	(1.61)	(1.88)	(2.48)	(3.54)	(2.08)	(2.60)
3-factor alpha	-0.21	-0.20	0.07	0.03	-0.13	0.05	0.06	0.16	0.34	0.25	0.47
	(-1.37)	(-1.70)	(0.67)	(0.36)	(-1.35)	(0.48)	(0.58)	(1.43)	(2.57)	(1.23)	(1.69)
4-factor alpha	-0.24	-0.25	0.07	0.02	-0.10	0.14	0.16	0.29	0.48	0.65	0.88
	(-1.49)	(-2.07)	(0.65)	(0.22)	(-1.00)	(1.34)	(1.56)	(2.72)	(3.84)	(3.77)	(3.54)

Table 4: Robustness Checks on 4-Factor Alphas of Portfolios Sorted on Relative Model Value

Each month, we sort all stocks into deciles according to the ratio of the equity value implied by our valuation model to the actual equity value (Decile 1=most overvalued, Decile 10=most undervalued). The portfolios are value-weighted and held for one subsequent month. The table reports four-factor alphas where the factors are market, size, book-to-market, and momentum. All alphas are in percent per month and the corresponding t-statistics are in parentheses. The full sample period is 1983 to 2012. The full sample period is broken up three different ways into subsamples in Panel A. Recession and expansion periods are based on NBER recession dummy. The holding period is increased to 3, 6, 12, and 18 months in Panel B.

	1	2	3	4	5	6	7	8	9	10	10−1
Full sample	-0.24	-0.25	0.07	0.02	0.10	0.14	0.16	0.29	0.48	0.65	0.88
	(-1.49)	(-2.07)	(0.65)	(0.22)	(-1.00)	(1.34)	(1.56)	(2.72)	(3.84)	(3.77)	(3.54)
Panel A: Sub-samples											
January	0.17	-0.52	0.73	0.06	0.20	-0.07	0.01	0.03	0.03	0.92	0.76
	(0.18)	(-1.09)	(1.92)	(0.13)	(0.51)	(-0.20)	(0.03)	(0.07)	(0.07)	(1.02)	(0.50)
Non-January	-0.26	-0.23	-0.01	0.03	-0.09	0.09	0.12	0.24	0.47	0.48	0.74
	(-1.66)	(-1.83)	(-0.05)	(0.31)	(-0.90)	(0.87)	(1.12)	(2.17)	(3.56)	(2.86)	(3.06)
Recession	-0.94	-0.17	0.01	0.22	0.19	0.38	0.46	0.57	1.10	0.79	1.73
	(-1.83)	(-0.44)	(0.02)	(0.79)	(0.50)	(1.00)	(1.04)	(1.28)	(2.37)	(1.17)	(1.94)
Expansion	-0.18	-0.20	0.07	0.01	-0.15	0.07	0.12	0.26	0.42	0.68	0.86
	(-1.05)	(-1.57)	(0.55)	(0.12)	(-1.46)	(0.71)	(1.14)	(2.41)	(3.16)	(3.76)	(3.23)
1980s	-0.57	-0.45	0.05	0.02	-0.15	-0.37	-0.10	0.24	0.30	0.14	0.70
	(-1.68)	(-1.67)	(0.15)	(0.13)	(-0.71)	(-1.70)	(-0.45)	(1.12)	(1.32)	(0.33)	(1.36)
1990s	-0.25	-0.14	0.05	0.01	-0.16	0.05	0.16	0.08	0.51	0.66	0.91
	(-1.05)	(-0.92)	(0.35)	(0.09)	(-1.09)	(0.32)	(1.05)	(0.55)	(2.33)	(2.79)	(2.52)
2000s	-0.23	-0.21	0.16	0.18	0.08	0.44	0.36	0.57	0.62	0.80	1.03
	(-0.93)	(-1.07)	(0.88)	(1.26)	(0.55)	(2.81)	(2.07)	(3.20)	(3.27)	(2.84)	(2.57)
Panel B: Longer Horizon											
3 months	-0.25	-0.20	0.06	0.03	-0.07	0.14	0.13	0.31	0.45	0.52	0.77
	(-1.59)	(-1.73)	(0.68)	(0.30)	(-0.82)	(1.57)	(1.34)	(3.27)	(3.80)	(3.44)	(3.32)
6 months	-0.23	-0.15	0.05	0.07	-0.05	0.12	0.14	0.29	0.43	0.50	0.73
	(-1.55)	(-1.37)	(0.50)	(0.81)	(-0.58)	(1.44)	(1.58)	(3.26)	(3.89)	(3.53)	(3.29)
12 months	(-0.26)	(-0.08)	(0.08)	(0.06)	(-0.02)	(0.11)	(0.16)	(0.24)	(0.36)	(0.39)	0.65
	(-1.83)	(-0.84)	(0.89)	(0.67)	(-0.20)	(1.27)	(1.85)	(2.92)	(3.55)	(2.91)	(3.12)
18 months	-0.26	-0.06	0.06	0.07	-0.02	0.09	0.15	0.22	0.29	0.32	0.57
	(-1.93)	(-0.59)	(0.74)	(0.89)	(-0.22)	(1.15)	(1.77)	(2.80)	(2.84)	(2.41)	(2.87)

Table 5: 4-Factor Alphas of Portfolios Double Sorted on Relative Model Value and Stock Characteristics

Each month, we first sort all stocks into quintiles based on a stock characteristic. The stocks are then further sorted into quintiles according to the ratio of the equity value implied by our valuation model to the actual equity value (R1=most overvalued, R5=most undervalued). The variable for the first sort is distress in Panel A, size in Panel B, leverage in Panel C, and stock return idiosyncratic volatility in Panel D. Distress is calculated based on CHS (2008) using current market data and quarterly accounting data of the previous quarter. Size is the market caitalization. Market leverage is calculated as the difference between financial liabilities (FL) and financial assets (FA) divided by equity market value. FL is equal to the sum of long-term debt, debt in current liabilities, carrying value of preferred stock, and preferred dividends in arrears, less preferred treasury stock. FA is cash and short-term investments. We assume a lag of four months for the availability of accounting statements. Idiosyncratic volatility is calculated as the standard deviation of the residual of regression of daily stock returns on the daily four factors over the last month. The holding period for all portfolios is one month and the portfolios are value-weighted. The table reports four-factor alphas where the factors are market, size, book-to-market, and momentum. All alphas are in percent per month and the corresponding t-statistics are in parentheses. The last column within each panel gives the fraction of value coming from the default option (in percent). To compute this fraction we run the model while shutting down the default option (i.e. imposing a restriction that the firm is always run by equityholders). The value of the option to default is then given by the difference in equity values with and without this option. The sample period is 1983 to 2012.

Panel A: Distress

	R1	R2	R3	R4	R5	R5−R1	DefOpt
D1	0.18	0.14	0.17	0.08	0.41	0.23	21.3
	(1.41)	(1.20)	(1.43)	(0.60)	(2.68)	(1.25)	
D2	-0.27	0.04	0.22	0.31	0.34	0.61	20.9
	(-2.04)	(0.31)	(1.96)	(2.42)	(2.63)	(3.25)	
D3	-0.27	0.02	-0.04	0.47	0.52	0.79	25.7
	(-1.68)	(0.11)	(-0.40)	(3.65)	(3.36)	(3.34)	
D4	-0.51	-0.51	-0.30	0.04	0.30	0.81	28.9
	(-2.07)	(-2.62)	(-1.85)	(0.20)	(1.36)	(2.45)	
D5	-1.08	-0.96	-0.27	0.54	0.24	1.32	42.5
	(-3.02)	(-2.89)	(-0.80)	(1.59)	(0.49)	(2.38)	

Panel B: Size

	R1	R2	R3	R4	R5	R5−R1	DefOpt
S1	-0.34	0.06	0.11	0.30	0.33	0.66	31.9
	(-2.23)	(0.58)	(1.22)	(2.76)	(2.19)	(3.66)	
S2	-0.31	-0.09	0.09	0.19	0.24	0.55	25.6
	(-2.10)	(-0.88)	(0.86)	(2.03)	(1.72)	(2.86)	
S3	-0.25	-0.26	-0.07	0.21	0.35	0.60	23.0
	(-1.93)	(-2.24)	(-0.68)	(1.84)	(2.72)	(3.13)	
S4	-0.17	0.05	-0.04	0.05	0.45	0.62	21.1
	(-1.18)	(0.46)	(-0.38)	(0.51)	(3.59)	(3.29)	
S5	-0.14	-0.01	0.00	-0.03	0.34	0.48	21.3
	(-1.06)	(-0.06)	(-0.01)	(-0.34)	(2.79)	(2.44)	

Panel C: Leverage

	R1	R2	R3	R4	R5	R5−R1	DefOpt
L1	0.23	0.73	0.21	0.76	1.02	0.80	19.7
	(0.95)	(3.40)	(1.16)	(3.78)	(3.83)	(2.26)	
L2	0.28	0.04	0.35	0.33	0.41	0.12	20.1
	(1.06)	(0.25)	(2.09)	(2.19)	(2.10)	(0.37)	
L3	-0.60	-0.20	0.09	0.07	0.35	0.94	23.6
	(-2.97)	(-1.44)	(0.75)	(0.53)	(2.15)	(3.70)	
L4	-0.15	-0.06	0.18	0.29	0.67	0.82	21.9
	(-0.92)	(-0.44)	(1.21)	(1.94)	(3.43)	(3.64)	
L5	-0.12	0.08	0.10	0.51	0.52	0.63	33.2
	(-0.64)	(0.52)	(0.58)	(2.45)	(1.49)	(1.62)	

Panel D: Idiosyncratic Volatility

	R1	R2	R3	R4	R5	R5−R1	DefOpt
IV1	-0.08	0.02	0.07	0.22	0.41	0.50	19.7
	(-0.70)	(0.18)	(0.65)	(2.12)	(3.16)	(2.69)	
IV2	-0.10	0.08	0.00	0.07	0.46	0.57	23.0
	(-0.77)	(0.66)	(-0.01)	(0.47)	(3.11)	(2.71)	
IV3	-0.06	-0.05	0.01	0.21	0.87	0.93	25.1
	(-0.30)	(-0.29)	(0.05)	(1.17)	(4.29)	(3.17)	
IV4	-0.54	-0.36	0.16	0.45	0.25	0.79	29.2
	(-2.01)	(-1.56)	(0.67)	(1.95)	(0.94)	(2.22)	
IV5	-0.98	-0.28	-0.60	-0.29	0.27	1.25	33.9
	(-2.57)	(-0.86)	(-2.25)	(-0.93)	(0.67)	(2.36)	

Table 6: 4-Factor Alphas of Portfolios Double Sorted on Relative Model Value (without the default option) and Stock Characteristics

Each month, we first sort all stocks into quintiles based on a stock characteristic. The stocks are then further sorted into quintiles according to the ratio of the equity value implied by our valuation model to the actual equity value (R1=most overvalued, R5=most undervalued). The variable for the first sort is distress, size, leverage, or stock return idiosyncratic volatility. Distress is calculated based on CHS (2008) using current market data and quarterly accounting data of the previous quarter. Size is the market caitalization. Market leverage is calculated as the difference between financial liabilities (FL) and financial assets (FA) divided by equity market value. FL is equal to the sum of long-term debt, debt in current liabilities, carrying value of preferred stock, and preferred dividends in arrears, less preferred treasury stock. FA is cash and short-term investments. We assume a lag of four months for the availability of accounting statements. Idiosyncratic volatility is calculated as the standard deviation of the residual of regression of daily stock returns on the daily four factors over the last month. We run the model two times, one time with default option and another time without the default option. The shutting down of the default option is accomplished by imposing a restriction that the firm is always run by equityholders. The holding period for all portfolios is one month and the portfolios are value-weighted. For each characteristic, the left column shows the 4-factor alpha of the long-short relative value portfolio R5−R1 as in Table 5. The right column shows the equivalent alphas based on model values without the option to default. The factors are market, size, book-to-market, and momentum. All alphas are in percent per month and the corresponding *t*-statistics are in parentheses. The sample period is 1983 to 2012.

	Distress		Size		Leverage		Volatility	
	with option	without option	with option	without option	with option	without option	with option	without option
Q1	0.23	0.11	0.66	0.37	0.80	0.65	0.50	0.61
	(1.25)	(0.51)	(3.66)	(2.00)	(2.26)	(1.47)	(2.69)	(3.29)
Q2	0.61	0.73	0.55	0.74	0.12	0.20	0.57	0.34
	(3.25)	(3.46)	(2.86)	(3.20)	(0.37)	(0.58)	(2.71)	(1.46)
Q3	0.79	0.79	0.60	0.88	0.94	0.77	0.93	0.68
	(3.34)	(3.28)	(3.13)	(4.63)	(3.70)	(3.04)	(3.17)	(2.14)
Q4	0.81	0.08	0.62	0.51	0.82	0.68	0.79	0.65
	(2.45)	(0.21)	(3.29)	(2.56)	(3.64)	(2.52)	(2.22)	(1.43)
Q5	1.32	0.57	0.48	0.42	0.63	0.90	1.25	0.43
	(2.38)	(0.75)	(2.44)	(2.15)	(1.62)	(2.41)	(2.36)	(0.75)

Table 7: Fama-MacBeth Regressions on Relative Model Value

We run cross-sectional Fama and MacBeth (1973) regressions each month of excess stock returns. The independent variables are (log) market capitalization, (log) market-to-book, past six-month return, distress-risk measure, leverage, idiosyncratic volatility, and relative model value. Market-to-book ratio is calculated as the ratio of current market value divided by book value of the previous quarter. We skip one month in calculating the six-month returns. Distress is calculated based on CHS (2008) using current market data and quarterly accounting data of the previous quarter. Market leverage is calculated as the difference between financial liabilities (FL) and financial assets (FA) divided by equity market value. FL is equal to the sum of long-term debt, debt in current liabilities, carrying value of preferred stock, and preferred dividends in arrears, less preferred treasury stock. FA is cash and short-term investments. We assume a lag of four months for the availability of accounting statements. Idiosyncratic volatility is calculated as the standard deviation of the residual of regression of daily stock returns on the daily four factors over the last month. Relative model value is the log of the ratio of the equity value implied by our valuation model to the actual equity value. All coefficients are multiplied by 100 and Newey-West corrected t-statistics (with six lags) are in parentheses. The sample period is 1983 to 2012.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Cnst	1.490	0.842	1.523	0.331	1.677	1.940	2.397
	(1.90)	(2.62)	(1.92)	(0.29)	(1.88)	(2.17)	(5.06)
Log(size)	-0.061		-0.064	-0.133	-0.115	-0.087	-0.113
	(-1.27)		(-1.30)	(-3.95)	(-2.58)	(-1.59)	(-3.38)
Log(MB)	-0.166		-0.097	-0.165	-0.215	-0.172	-0.085
	(-2.69)		(-1.82)	(-3.29)	(-4.12)	(-3.48)	(-1.79)
Past return	0.276		0.306	0.137	0.309	0.286	0.354
	(1.47)		(1.67)	(0.78)	(1.73)	(1.64)	(2.01)
Relative model value		0.216	0.193	0.555	0.325	0.223	0.065
		(4.36)	(4.22)	(4.48)	(3.60)	(3.66)	(1.04)
Distress				-0.267			
				(-2.83)			
Distress × Relative model value				0.062			
				(3.78)			
Log(size) × Relative model value					-0.044		
					(-2.72)		
Leverage						-0.213	
						(-4.17)	
Leverage × Relative model value						0.017	
						(0.92)	
Idio volatility							-14.056
							(-2.01)
Idio volatility × Relative model value							3.095
							(1.95)

Table 8: 4-Factor Alphas of Portfolios Sorted on Value Gap and Anomaly Variable

Each month, we sort all stocks into terciles based on value gap. Value gap is defined as the absolute value of the log of the ratio of the equity value implied by our valuation model and the actual equity value. Portfolio VG1 has the least value gap while portfolio VG3 has the highest value gap. The stocks are then sorted (the second sort is a sequential sort) into quintiles based on an anomaly variable. Panel A uses distress (quintiles D1 to D5), Panel B uses leverage (quintiles L1 to L5), and Panel C uses idiosyncratic volatility (quintiles IV1 to IV5). Distress is calculated based on CHS (2008) using current market data and quarterly accounting data of the previous quarter. Market leverage is calculated as the difference between financial liabilities (FL) and financial assets (FA) divided by equity market value. FL is equal to the sum of long-term debt, debt in current liabilities, carrying value of preferred stock, and preferred dividends in arrears, less preferred treasury stock. FA is cash and short-term investments. We assume a lag of four months for the availability of accounting statements. Idiosyncratic volatility is calculated as the standard deviation of the residual of regression of daily stock returns on the daily four factors over the last month. The table reports four-factor alphas where the factors are market, size, book-to-market, and momentum. All alphas are in percent per month and the corresponding t-statistics are in parentheses. The sample period is 1983 to 2012.

Panel A: Distress	VG1	VG2	VG3
D1	0.14	0.11	0.27
	(1.22)	(0.94)	(2.04)
D2	0.17	0.14	-0.08
	(1.55)	(1.46)	(-0.61)
D3	0.08	0.19	-0.02
	(0.72)	(1.64)	(-0.10)
D4	0.04	0.04	-0.67
	(0.35)	(0.29)	(-2.75)
D5	-0.42	-0.64	-0.56
	(-2.16)	(-2.96)	(-1.55)
D1−D5	0.56	0.75	0.82
	(2.38)	(2.92)	(2.18)

Panel B: Leverage	VG1	VG2	VG3
L1	0.40	0.63	0.51
	(2.11)	(3.78)	(2.40)
L2	0.43	0.27	0.11
	(3.02)	(1.74)	(0.52)
L3	0.08	-0.06	-0.09
	(0.68)	(-0.55)	(-0.48)
L4	0.03	0.07	0.10
	(0.28)	(0.48)	(0.57)
L5	-0.05	-0.13	0.37
	(-0.38)	(-0.93)	(1.64)
L1−L5	0.45	0.76	0.15
	(2.00)	(3.55)	(0.48)

Panel C: Idiosyncratic volatility	VG1	VG2	VG3
IV1	0.10	0.11	0.09
	(1.04)	(1.05)	(0.75)
IV2	0.15	0.05	0.02
	(1.40)	(0.52)	(0.13)
IV3	-0.15	0.07	-0.01
	(-1.10)	(0.44)	(-0.07)
IV4	0.15	0.06	-0.39
	(0.80)	(0.35)	(-1.32)
IV5	-0.23	-0.45	-0.99
	(-1.04)	(-2.08)	(-3.08)
IV1−IV5	0.34	0.55	1.08
	(1.40)	(2.25)	(3.10)

Table 9: Fama-MacBeth Regressions on Value Gap and Anomaly Variable

We run cross-sectional Fama and MacBeth (1973) regressions each month on excess stock returns. The independent variables are (log) market capitalization, (log) market-to-book, past six-month return, distress-risk measure, leverage, idiosyncratic volatility, and value gap. Value gap is defined as the absolute value of the log of the ratio of the equity value implied by our valuation model and the actual equity value. Distress is calculated based on CHS (2008) using current market data and quarterly accounting data of the previous quarter. Market leverage is calculated as the difference between financial liabilities (FL) and financial assets (FA) divided by equity market value. FL is equal to the sum of long-term debt, debt in current liabilities, carrying value of preferred stock, and preferred dividends in arrears, less preferred treasury stock. FA is cash and short-term investments. We assume a lag of four months for the availability of accounting statements. Idiosyncratic volatility is calculated as the standard deviation of the residual of regression of daily stock returns on the daily four factors over the last month. Panel B shows the coefficients on the interaction terms when we estimate value gap from a model without the default option. The shutting down of the default option is accomplished by imposing a restriction that the firm is always run by equityholders. All coefficients are multiplied by 100 and Newey-West corrected t-statistics (with six lags) are in parentheses. The sample period is 1983 to 2012.

Panel A: With the default option						
	(1)	(2)	(3)	(4)	(5)	(6)
Cnst	-2.279	-1.244	2.102	1.395	1.262	2.056
	(-2.42)	(-1.30)	(1.38)	(1.91)	(1.50)	(4.69)
Log(size)	-0.108	-0.033	-0.148	-0.040	0.008	-0.076
	(-3.74)	(-3.27)	(-1.02)	(-0.91)	(0.10)	(-2.51)
Log(MB)	-0.208	-0.199	-0.109	-0.210	-0.091	-0.098
	(-3.83)	(-3.99)	(-0.78)	(-4.46)	(-1.74)	(-2.18)
Past return	0.286	0.242	0.504	0.435	0.585	0.563
	(1.66)	(1.26)	(2.84)	(2.34)	(3.40)	(3.10)
Value gap		-0.500		-0.165		-0.006
		(-3.61)		(-3.18)		(-0.09)
Distress	-0.552	-0.408				
	(-6.71)	(-5.05)				
Distress $\times$ Value gap		-0.065				
		(-3.49)				
Leverage			-0.209	-0.252		
			(-5.37)	(-4.37)		
Leverage $\times$ Value gap				0.041		
				(1.57)		
Idio volatility					-27.320	-19.341
					(-3.37)	(-2.60)
Idio volatility $\times$ Value gap						-4.504
						(-2.23)
Panel B: Without the default option						
Distress $\times$ Value gap		-0.059				
		(-1.42)				
Leverage $\times$ Value gap				0.105		
				(2.00)		
Idio volatility $\times$ Value gap						-1.884
						(-0.53)